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MHD Flow Past a Parabolic Started Isothermal Vertical Plate with Variable Mass Diffusion in the Presence of Chemical Reaction of First Order

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Abstract

Laplace transform solution of MHD flow past a parabolic starting motion of the infinite isothermal vertical plate with variable mass diffusion, in the presence of homogeneous chemical reaction of first order has been studied. The plate temperature is raised uniformly and the concentration levels near the plate are raised linearly with time. The effect of velocity, temperature and concentration profiles are studied for different physical parameters like chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number and time. It is observed that the velocity increases with increasing values the thermal Grashof number or mass Grashof number. The trend is just reversed with respect to the chemical reaction parameter as well as magnetic field parameter.

Keywords: parabolic, homogeneous, heat and mass transfer, chemical reaction, isothermal, first order, vertical plate, magnetic field..

Introduction

MHD plays an important role in petroleum industries, geophysics and in astrophysics. It also finds applications in many engineering problems such as magnetohydrodynamic generator, plasma studies, in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has applications in metrology, solar physics and in the movement of earth's core. It has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics.

Chemical reactions can be divided in to two groups. They are (i) Homogeneous and (ii) Heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration of only one reactant and is independent of others. Decomposition of nitrogen pent oxide in the gas phase as well in an organic solvent like CCl_4, conversion of N-chloroacetanilide into p- chloroacetanilide, hydrolysis of methyl acetate and inversion of cane sugar. The radioactive disintegration of unstable nuclei are the best examples of first order reactions.

Chambre and Young [3] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das et al [4] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al [5]. The dimensionless governing equations were solved by the usual Laplace-transform technique.

Pure heat transfer effects on impulsively started an infinite vertical plate in the presence of magnetic field was studied by Soundalgekar et al [11]. Again, mass transfer effects on MHD flow past an impulsively started an infinite isothermal vertical plate with uniform mass diffusion studied by Soundalgekar et al [12]. Rajesh Kumar et al [8] have studied exact solution of hydromagnetic flow on moving vertical surface with prescribed uniform heat flux. The effect of viscous dissipation on Darcy free convection flow

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over a vertical plate with an exponential temperature was analyzed by Magyari and Rees [6]. The combined effects of heat and mass transfer along a vertical plate in the presence of a transverse magnetic field were studied by Ramesh Babu, and Shankar [10]. Rajput & Kumar [9] studied the magnetic field effects on flow past an impulsively started vertical plate with variable temperature and mass diffusion. Recently, Muthucumarswamy et al [7] studied MHD effects on accelerated isothermal vertical plate with uniform mass diffusion using Laplace transform method.

Agrawal et al [1] studied free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic filed. Agrawal et al [2] further extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate with transverse magnetic plate. The governing equations are tackled using Laplace transform technique

It is proposed to study the effects of on flow past an infinite isothermal vertical plate subjected to parabolic motion with variable mass diffusion, in the presence of magnetic field and chemical reaction of first order. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

Mathematical Analysis

The unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical plate with variable mass diffusion, in the presence of chemical reaction of first order has been considered. The x' -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞ .At time $t' > 0$, the plate is started with a velocity $u = u_0\, t'^2$ in its own plane against gravitational field. The temperature from the plate is raised to T_w and the concentration level near the plate are also raised to C'_w. A chemically reactive species which transforms according to a simple reaction involving the conecntration is emitted from the plate and diffuses into the fluid. The plate is also subjected to a uniform magnetic field of strength B_0 is assumed to be applied normal to the plate. The reaction is assumed to take place entirely in the stream. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \qquad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} \qquad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - k_l(C' - C'_\infty) \qquad (3)$$

With the following initial and boundary conditions:

$$u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all } y, t' \leq 0$$

$$t' > 0: u = u_0 . t'^2, \ T = T_w, \ C' = C'_\infty + (C'_w - C'_\infty)At', \quad \text{at} \quad y = 0 \qquad (4)$$

$$u \to 0 \quad T \to T_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y \to \infty$$

On introducing the following non-dimensional quantities:

$$U = u\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad t = \left(\frac{u_0^{\,2}}{\nu}\right)^{1/3} t', \quad Y = y\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \ C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$

$$Gr = \frac{g\beta(T_w - T_\infty)}{(\nu u_0)^{1/3}},\ Gc = \frac{g\beta(C'_w - C'_\infty)}{(\nu u_0)^{1/3}},\ K = K_l\left(\frac{\nu}{u_0^{\,2}}\right)^{\frac{1}{3}},\ \Pr = \frac{\mu C_p}{k},\ \ Sc = \frac{\nu}{D} \tag{5}$$

$$M = \frac{\sigma B_0^2}{\rho}\left(\frac{\nu}{u_0^2}\right)^{1/3}$$

The equations (1) to (3) reduces to the following dimensionless form:

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} - MU \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \tag{8}$$

The corresponding initial and boundary conditions in non-dimensionless form are as follows:

$$\begin{aligned} &U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } Y, t \le 0 \\ t > 0: \quad &U = t^2, \quad \theta = 1, \quad C = t \quad \text{at } Y = 0 \\ &U \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as } Y \to \infty \end{aligned} \tag{9}$$

The dimensionless governing equations (6) to (8) and the corresponding initial and boundary conditions (9) are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = erfc(\eta\sqrt{\Pr}) \tag{10}$$

$$\begin{aligned} C = &\frac{t}{2}\left[\exp\left(2\eta\sqrt{Sc\,K\,t}\right)erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{Sc\,K\,t}\right)erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right)\right] \\ &- \frac{\eta\sqrt{Sc}\sqrt{t}}{2\sqrt{K}}\left[\exp\left(-2\eta\sqrt{Sc\,K\,t}\right)erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) - \exp\left(2\eta\sqrt{Sc\,K\,t}\right)erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right)\right] \end{aligned} \tag{11}$$

$$\begin{aligned} U = &\left[\frac{\left(\eta^2 + M\,(t + 2bd)\right)t + 2M(c + d)}{2M}\right]\left[\exp\left(2\eta\sqrt{M\,t}\right)erfc\left(\eta + \sqrt{M\,t}\right) + \exp\left(-2\eta\sqrt{M\,t}\right)erfc\left(\eta - \sqrt{M\,t}\right)\right] \\ &+ \left[\frac{\eta\sqrt{t}\left(1 - 4M\,(t + bd)\right)}{4M^{3/2}}\right]\left[\exp\left(-2\eta\sqrt{M\,t}\right)erfc\left(\eta - \sqrt{M\,t}\right) - \exp\left(2\eta\sqrt{M\,t}\right)erfc\left(\eta + \sqrt{M\,t}\right)\right] \\ &- \frac{\eta t}{M\sqrt{\pi}}\exp(-(\eta^2 + M\,t)) \quad - 2cerfc(\eta\sqrt{\Pr}) \\ &+ \frac{bd\eta\sqrt{Sc}\sqrt{t}}{\sqrt{K}}\left[\exp\left(-2\eta\sqrt{Sc\,K\,t}\right)erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right) - \exp\left(2\eta\sqrt{Sc\,K\,t}\right)erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right)\right] \\ &\quad - d(bt + 1)\left[\exp\left(2\eta\sqrt{Sc\,K\,t}\right)erfc\left(\eta\sqrt{Sc} + \sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{Sc\,K\,t}\right)erfc\left(\eta\sqrt{Sc} - \sqrt{Kt}\right)\right] \end{aligned}$$

$$+d\exp(bt)\left[\exp\left(2\eta\sqrt{Sc(K+b)t}\right)erfc\left(\eta\sqrt{Sc}+\sqrt{(K+b)t}\right)+\exp\left(-2\eta\sqrt{Sc(K+b)t}\right)erfc\left(\eta\sqrt{Sc}-\sqrt{(K+b)t}\right)\right]$$

$$-d\exp(bt)\left[\exp\left(2\eta\sqrt{(M+b)t}\right)erfc\left(\eta+\sqrt{(M+b)t}\right)+\exp\left(-2\eta\sqrt{(M+b)t}\right)erfc\left(\eta-\sqrt{(M+b)t}\right)\right]$$
$$-c\exp(at)\left[\exp\left(2\eta\sqrt{(M+a)t}\right)erfc\left(\eta+\sqrt{(M+a)t}\right)+\exp\left(-2\eta\sqrt{(M+a)t}\right)erfc\left(\eta-\sqrt{(M+a)t}\right)\right]$$
$$+c\exp(at)\left[\exp\left(2\eta\sqrt{\Pr at}\right)erfc\left(\eta\sqrt{\Pr}+\sqrt{at}\right)+\exp\left(-2\eta\sqrt{\Pr at}\right)erfc\left(\eta\sqrt{\Pr}-\sqrt{at}\right)\right] \quad (12)$$

Where, $a=\dfrac{M}{\Pr-1}$, $b=\dfrac{M-K\,Sc}{Sc-1}$, $c=\dfrac{Gr}{2a(1-\Pr)}$, $d=\dfrac{Gc}{2b^2(1-Sc)}$ and $\eta=\dfrac{Y}{2\sqrt{t}}$.

Results and Discussion

For physical understanding of the problem numerical computations are carried out for different physical parameters K, M, Pr, Gr, Gc, Sc and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the values of Prandtl number Pr are chosen such that they represent water (Pr = 7.0). The numerical values of the velocity are computed for different physical parameters like chemical reaction parameter, magnetic field parameter, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

Figure 1 represents the effect of the concentration profiles for different values of the chemical reaction parameter ($K=0.2,2,5,10$) at $t=0.4$. The effect of chemical reaction parameter is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the concentration increases with decreasing chemical reaction parameter.

The velocity profiles for different values of the chemical reaction parameter $(K=5,8,15)$, $Gr=5=Gc, \Pr=7, M=2$ and $t=0.6$ are shown in figure 2. It is observed that the velocity increases with decreasing chemical reaction parameter.

Figure 3 demonstrates the effect of velocity for different values of the magnetic field parameter $(M=1.2,1.6,2)$, $Gr=5=Gc, \Pr=7, K=8$ and $t=0.6$. It was observed that the velocity increases with decreasing values of the magnetic field parameter. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

Figure 4 demonstrates the effects of different thermal Grashof number (*Gr* = 2, 5), mass Grashof number (*Gc* = 5, 10), K = 8, M = 2 and Pr = 7 on the velocity at *t* = 0.6. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number. The trend shows that the influence of mass Grashof number is predominant than thermal Grashof number.

The velocity profiles for different values of the time $(t=0.2,0.4,0.6,0.8)$, $Gr=5=Gc$, $K=8$ and $M=2$ are presented in figure 5. The trend shows that the velocity increases with incrasing values of the time t. The effect of velocity profiles for different values of the Schmidt number $(Sc=0.16,0.3,0.6), Gr=5=Gc, \Pr=7, M=4$ and $t=0.6$ are shown in figure 6. It is observed that the velocity increases with decrasing values of the Schmidt number.

Conclusion

An exact analysis is performed to study the MHD flow past a parabolic started an infinite isothermal vertical plate with variable mass diffusion, in the presence of chemical reaction of first order. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of the temperature, the concentration and the velocity fields for different physical parameters like chemical reaction parameter, magnetic field parameter, thermal Grashof number, mass Grashof number and *t* are studied graphically. The conclusions of the study are as follows:

(i) The velocity increases with increasing thermal Grashof number or mass Grashof number and time *t* in the presence of magnetic field parameter. But the trend is just reversed with respect to the chemical reaction parameter or

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magnetic field parameter.

(ii) The temperature of the plate increases with decreasing values of the Prandtl number

(iii) The plate concentration increases with decreasing values of the chemical reaction parameter.

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Nomenclature

A	Constants
C'	species concentration in the fluid $kg\,m^{-3}$
C	dimensionless concentration
C_p	specific heat at constant pressure $J.kg^{-1}.k$
D	mass diffusion coefficient $m^2.s^{-1}$
Gc	mass Grashof number
Gr	thermal Grashof number
g	acceleration due to gravity $m.s^{-2}$
k	thermal conductivity $W.m^{-1}.K^{-1}$
Pr	Prandtl number
Sc	Schmidt number
T	temperature of the fluid near the plate K
t'	time s
u	velocity of the fluid in the x'-direction $m.s^{-1}$
u_0	velocity of the plate $m.s^{-1}$
u	dimensionless velocity
y	coordinate axis normal to the plate m
Y	dimensionless coordinate axis normal to the plate

Greek symbols

β	volumetric coefficient of thermal expansion K^{-1}

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β^* volumetric coefficient of expansion with concentration K^{-1}
μ coefficient of viscosity $Ra.s$
ν kinematic viscosity $m^2.s^{-1}$
ρ density of the fluid $kg.m^{-3}$
τ dimensionless skin-friction $kg.m^{-1}.s^2$
θ dimensionless temperature
η similarity parameter
$erfc$ complementary error function

Subscripts

w conditions at the wall
∞ free stream conditions

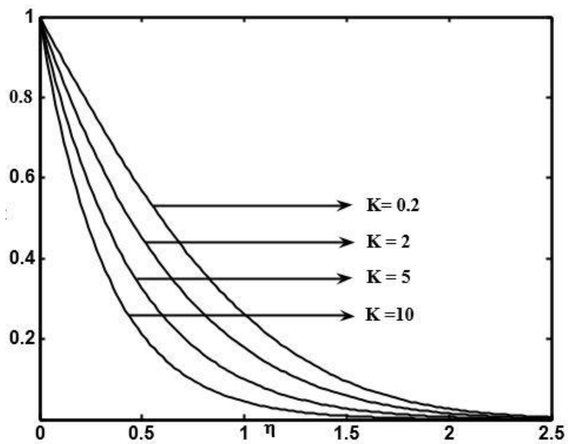

Fig. 1: Concentration profiles for different values of K

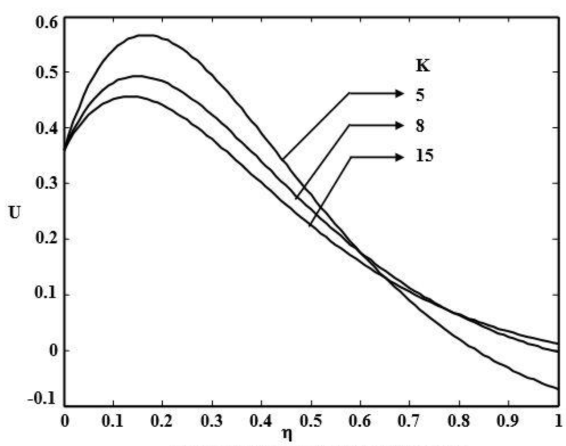

Fig. 2 Velocity profiles for different K

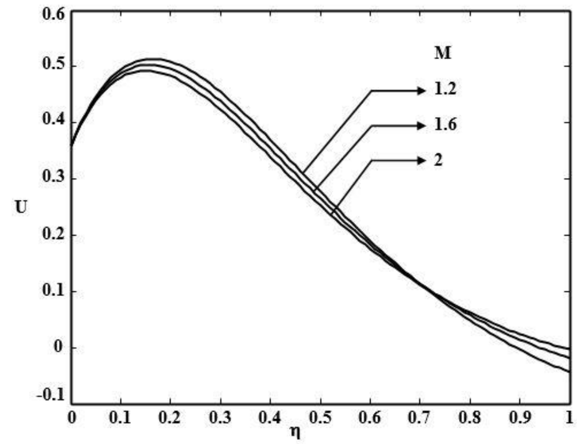

Fig. 3 Velocity profiles for different M

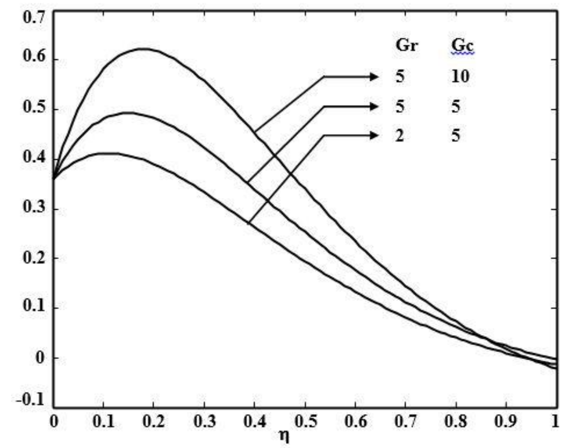

Fig. 4 Velocity profiles for different Gr & Gc

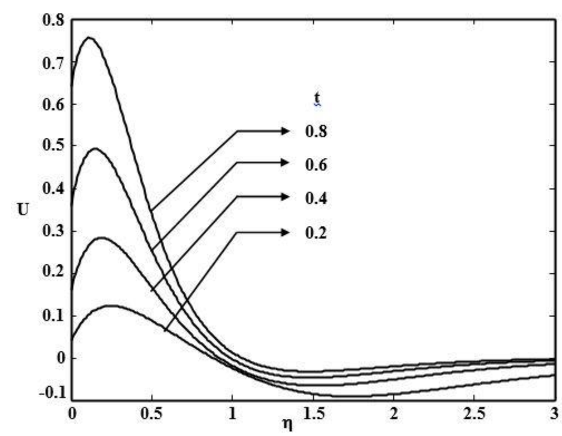

Fig. 5 Velocity profiles for different t

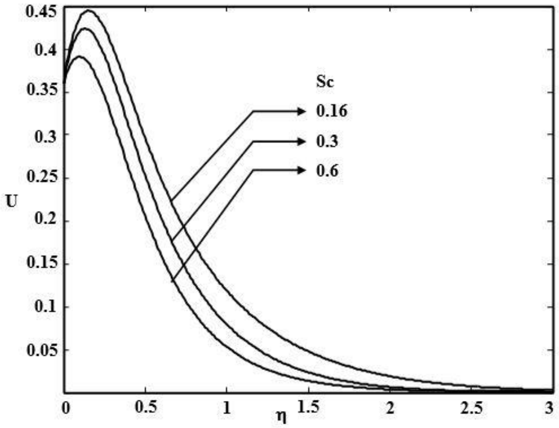

Fig. 6 Velocity profiles for different Sc